Pay-as-you-go Pension System and Externalities between Communities in a Model with Endogenous Longevity

-Moral hazard and Adverse Selection-

Nobuo Akai*

Institute of Economic Research

Kobe University of Commerce

This Version: 99/10/26

*Correspondence to: Nobuo Akai, Institute of Economic Research, Kobe University of Commerce, 8-2-1 Gakuen-Nishimachi, Nishi-ku, Kobe, 651-21, Japan, TEL:+81-(0)78-794-6161, FAX:+81-(0)78-794-6166, E-mail: akai@kobeuc.ac.jp

Abstract

In order to examine problems created by the heterogeneity of the lifetime among communities, we develop the model with endogenous longevity. First, we derive that, in the competitive equilibrium, moral hazard problem and adverse selection problem occur and they result in inefficient allocation. Second, in order to remove inefficiency, we introduce the pay-as-you-go public pension system. We derive the possibility that the lump-sum pension system improves efficiency of the economy. In addition, it is shown that the finance system based on two types of taxes is needed to achieve the efficient allocation.

1. Introduction

As the lifetime is getting longer, the role of the pension system becomes important. The difference of the lifetime affects the future life of consumers through the pension payments. The difference of the degree of longevity among communities is created not only by the original difference of the population distribution but also endogenously by the inter-temporal utility maximization behavior of consumers. In other words, if the time preference of consumers for the second period is different among communities, then the population of the elderly people is different because the people with the high weight in the retirement life invest their health and increase their lifetime. Therefore when we consider the model with the health related investment, then the difference of length of lifetime is created endogenously.

In this type of economy, it is considered that there exist two inefficiencies. First is the externality effect related to the interest rate of money, which is regarded as the moral hazard problem. This inefficiency affects the relative price between health-related investment and consumption. Second is the externality effect among heterogeneous communities, which is regarded as the adverse selection problem. This inefficiency affects the relative price between the present period's consumption and the future period's consumption.

Based on this intuition, it is important to clarify what types of inefficiency is created in the economy with heterogeneous communities and endogenous longevity, and how the government removes this inefficiency, that is, how to introduce the public pension system.

Maeda and Akai (1998) developed the model with the endogenous interest rate of money and derived inefficiency related to the interest rate of money, based on Davis and Kuhn (1992) which discussed the model with the health-related investment affecting longevity. Also Maeda and Akai (1998) discussed how to finance the pension system.

This paper extends existing models by incorporating the heterogeneous communities and derives the following four new results. First, inefficiency derived by Maeda and Akai (1998) is summarized as the moral hazard problem. Second, it is found that there exists adverse selection problem related to the heterogeneity between communities. Third, it is derived that the lump-sum pension system which decreases adverse selection problem may be improve efficiency, which is different from the results derived in Maeda and Akai (1998) and Samuelson (1958). Forth, it is shown that the optimal methods of financing the pension system in the economy with heterogeneous communities is different from the ones in the simple economy with one homogeneous community, which is discussed in Maeda and Akai (1998).

This paper is organized as follows. In section 2, we develop the model. In section 3, efficient allocation is characterized as a benchmark of the allocation. In section 4, we derive the biological interest rate of money and the return from private pension in the competitive market, and show that both the interest rate and the return become equal. In section 5, a competitive equilibrium is characterized. In section 6, a competitive allocation is compared with an efficient allocation, from the view points of moral hazard and adverse selection. Section 7 discusses the optimal pension system toward an efficient allocation. Section 8 concludes this paper.

2. Model

We consider an overlapping generation model in which there exist two communities ($i = 1,2$) and the same constant numbers of agents are born in each community in each period. The population of newly born agents is normalized to one. Each agent lives for two periods at maximum, but may die at the end of the first period of his life. The utility of an agent born at t is given by $u(c_t^i(t)) + \beta^i E[u(c_t^i(t+1))]$. $c_t^i(s)$ represents his consumption of generation t in period s in community i. β^i is the reciprocal of the discount rate with $0 < \beta^i \leq 1$. Let $h^i(t+1)$ be the amount of health-related investment of the young agent of the t generation and $p(h^i(t+1))$ be the probability that he lives for two periods. Then his lifetime utility can be written as $u(c_t^i(t)) + \beta^i p(h^i(t+1))u(c_t^i(t+1))$. We assume that u and p are continuous, strictly increasing, strictly concave, twice continuously differentiable. There is only one good in the economy and each agent is endowed with w units of this good when young and nothing when old. In this section, we assume that this good is not storable, and is used for consumption or health-related investment. The feasibility condition for the economy is

$$c_t^i(t) + h^i(t+1) + p(h^i(t))c_{t-1}^i(t) = w^i$$

The range of p is set such that [0,1], and $u(.)p(.)$ is assumed to be strictly concave. In addition, we set the following boundary conditions: $\lim_{c\to 0} u'(c) = \infty, \lim_{h\to 0} p'(h^i) = \infty, u(0) > 0$ and $p(0) > 0$.

3. Efficient Allocation

In this paper, we only consider stationary allocations in the sense that $c_t^i(t) = c_0^i, c_{t-1}^i(t) = c_1^i$ and $h^i(t) = h^i$. Then the efficient allocation is the solution for the following problem.

$$\max_{c_0^i, c_1^i, h^i} u(c_0^i) + \beta^i p(h^i)u(c_1^i)$$

subject to

$$c_0^1 + h^1 + p(h^1)c_1^1 + c_0^2 + h^2 + p(h^2)c_1^2 = w^1 + w^2$$

$$u(c_0^{-i}) + \beta^{-i} p(h^{-i})u(c_1^{-i}) = \bar{u}^{-i}$$

Assuming that there exist interior solutions, the efficient allocation, $\bar{c}_0^i, \bar{c}_1^i, \bar{h}^i$, can be achieved such that the following first order conditions are satisfied.

$$\frac{u'(\bar{c}_0^i)}{u'(\bar{c}_1^i)} = \beta^i, \tag{1}$$

$$\frac{u(\bar{c}_1^i)}{u'(\bar{c}_0^i)} = \frac{1 + p'(\bar{h}^i)\bar{c}_1^i}{\beta^i p'(\bar{h}^i)}, \tag{2}$$

$$\bar{c}_0^1 + \bar{h}^1 + p(\bar{h}^1)\bar{c}_1^1 + \bar{c}_0^2 + \bar{h}^2 + p(\bar{h}^2)\bar{c}_1^2 = w^1 + w^2 \tag{3}$$

Equation (1) means that the marginal rate of substitution on the first period consumption with respect to the second period consumption is equal to the time preference parameter, β . Equation (2) means that the marginal utility by the marginal increase of the health related investment, $u(\bar{c}_1^i)\beta^i p'(\bar{h}^i)$, is equal to the marginal cost by the decrease of the first period consumption created by the marginal increase of the health related investment, $u'(\bar{c}_0^i)(1 + p'(\bar{h}^i)\bar{c}_1^i)$, which includes two effects; first is the direct effect on the first period consumption by the increase of the health related investment, $u'(\bar{c}_0^i)$, and second is the indirect effect on the first period consumption by the reduction of the budget through the longevity which the increase of the health related investment creates, $u'(\bar{c}_0^i)p'(\bar{h}^i)\bar{c}_1^i$. Finally, equation (3) is the budget constraint for the economy. These three equations have many solutions because these conditions are the necessary condition for the efficient resource allocation and inequality between two agents is not considered.

4. Biological Interest Rate of Money

Money (or Actuarial Notes) is valuable in the economy which young agents can demand money to transfer or save wealth to the second period. We assume that there exits $M^i(t)$ units of money in community i in period t . Let $q(t)$ be the price of money in terms of commodity and be common in both communities because of free mobility of money between communities. The young agents in community i pay $q(t)M^i(t)$ units of commodity to buy $M^i(t)$ units of money, and if they are alive in the second period, they can buy $q(t+1)M^i(t)$ units of commodity by using money he holds. We assume that if he dies at the end of period t , money is confiscated by the government and abandoned. Since the probability of being alive is $p(h^i(t+1))$, we have

$$M^1(t+1) + M^2(t+1) = p(h^1(t+1))M^1(t) + p(h^2(t+1))M^2(t).$$

When we focus on a stationary monetary equilibrium in the sense that young agents save the constant amount of commodity in each period in order to transfer them to the second period, that is, $q(t)(M^1(t) + M^2(t))$ is constant over time. Therefore we have

$$\begin{aligned} q(t)(M^1(t) + M^2(t)) &= q(t+1)(M^1(t+1) + M^2(t+1)) \\ &= q(t+1)(p(h^1(t+1))M^1(t) + p(h^2(t+1))M^2(t)). \end{aligned}$$

The change of the price of money becomes

$$\frac{q(t+1)}{q(t)} = \frac{M^1(t) + M^2(t)}{p(h^1(t))M^1(t) + p(h^2(t))M^2(t)}$$

This means that the one unit of money in period t has the value of

$$\frac{M^1(t) + M^2(t)}{p(h^1(t))M^1(t) + p(h^2(t))M^2(t)} > 1$$

in period $t+1$.

Under the existence of money, the budget constraint of consumers in community i becomes

$$c_t^i(t) + h^i(t) + l^i(t) + q(t)M^i(t) = w^i$$

$$c_t^i(t+1) = r(t+1)l^i(t) + q(t+1)M^i(t)$$

where $l^i(t)$ is private lending and borrowing, $r(t+1)$ is the interest rate for private lending and borrowing.

In a monetary equilibrium, from the arbitrage condition, the biological interest rate is derived as follows;

$$r(t+1) = \frac{q(t+1)}{q(t)} = \frac{M^1(t) + M^2(t)}{p(h^1(t))M^1(t) + p(h^2(t))M^2(t)} .$$

In this paper, we consider money. However the role of money is replaced by private pension. We can confirm that the same interest rate is derived in the economy with the private pension system.[1]

5. Competitive Allocation

Next, we characterize the competitive equilibrium based on the utility maximization behavior of

---

[1] In this paper, we have introduced money. However money may be regarded as private pension. In this footnote, we confirm that the same interest rate is derived in the economy with the private pension system. For simplicity, the private pension system is operated by the non-profit organization. Assume that M^i is the demand for private pension in community i and B^i is the benefit from the pension system in the second period. Then the budget constraint of the private pension system becomes

$$M^1(t) + M^2(t) = p(h^1(t))B^1(t+1) + p(h^2(t))B^2(t+1).$$

As long as consumers can buy the private pension system in another community, the return from the private pension system must be the same in the both communities. Defining the return from the private pension system as $r^{PP}(t+1) \equiv \frac{B^1(t+1)}{M^1(t)} = \frac{B^2(t+1)}{M^2(t)}$, the return becomes

$$r^{PP}(t+1) = \frac{M^1(t) + M^2(t)}{p(h^1(t))M^1(t) + p(h^2(t))M^2(t)},$$

which is equal to the interest rate of money.

consumers. Similar to section 3, we only consider allocations in the stationary equilibrium in the sense that $c_t^i(t) = c_0^i, c_{t-1}^i(t) = c_1^i, h^i(t) = h^i$ and $r(t+1) = r$.

Recently young people are interested in the effect of foods on their health and lifetime. However they have not cared the demerit created by the decrease of the interest rate, that is, the decrease of the payment of pension because of longevity of their lifetime. [2]Therefore, in this section we assume that the interest rate is given for the consumers.

Then the competitive allocation is the solution for the following problem.

$$\max_{c_0^i, c_1^i, h^i} u(c_0^i) + b^i p(h^i) u(c_1^i)$$

subject to

$$c_0^i + h^i + \frac{c_1^i}{r} = w^i$$

Therefore we can characterize a competitive equilibrium ($c_0^{i(M)}, c_1^{i(M)}, h^{i(M)}$) as follows;

$$\frac{u'(c_0^{i(M)})}{u'(c_1^{i(M)})} = b^i p(h^{i(M)}) r, \tag{7}$$

$$\frac{u(c_1^{i(M)})}{u'(c_0^{i(M)})} = \frac{1}{b^i p'(h^{i(M)})}, \tag{8}$$

$$c_0^{i(M)} + h^{i(M)} + \frac{c_1^{i(M)}}{r} = w^i . \tag{9}$$

$$r = \frac{w^1 - c_0^{1(M)} - h^{1(M)} + w^2 - c_0^{2(M)} - h^{2(M)}}{p(h^{1(M)})(w^1 - c_0^{1(M)} - h^{1(M)}) + p(h^{2(M)})(w^2 - c_0^{2(M)} - h^{2(M)})} \tag{10}$$

The intuition of each condition described above is follows. Equation (7) means that the marginal rate of substitution on the first period consumption with respect to the second period consumption is equal to the time preference parameter, b , adjusted by the interest rate and the possibility living in the second period. If communities are symmetric then the interest rate becomes equal to $\frac{1}{p(h^i)}$ and equation (7) reduces to equation (1). However the interest rate is determined by the behavior of both agents in both communities. This interaction between communities creates inefficiency. (This inefficiency is summarized in the next section more clearly.)

Next, equation (8) means that the marginal utility by the marginal increase of the health related investment, $u(\bar{c}_1^i) b^i p'(\bar{h}^i)$, is equal to the direct effect on the first period consumption by the increase of the health related investment, $u'(\bar{c}_0^i)$. Comparing with equation (2), we know that the indirect effect is not considered. The reason is that the agent does not care the reduction of the interest rate created by the increase of the health related investment. This distortion creates

[2] Though longevity and health investment are discussed closely, the relationship between health investment and pension is separately recognized in general.

inefficiency. (This inefficiency is summarized in the next section more clearly.)

Finally, equation (9) is the budget constraint for the agent in the community i. This is the enough condition to satisfy the necessary condition for the efficient allocation (equation (3)).

6. Comparison between Competitive Allocation and Efficient Allocation -Moral hazard and Adverse selection-

In this section, we compare the first order conditions of the efficient allocation with those of the competitive allocation. By comparing equations (1)-(3) with (7)-(10), we have the following result.

Proposition 1

In a monetary competitive equilibrium, the efficient allocation can not be achieved.

The reason why the efficient level can not be achieved in the competitive equilibrium is related to the following two inefficiencies.

Moral Hazard

First, in spite of the fact that the health-related investment affects the interest rate of money in the second period through the longevity of the lifetime, they do not consider this mechanism. This inefficiency affects the relative price between health-related investment and consumption. (Compare equation (2) with equation (8).)

When we regard money as private pension, this inefficiency can be easily considered as the *moral hazard* problem derived by the asymmetric information for the health investment. The reason is as follows. When the amount of the health investment is verifiable for the non profit organization which provides the private pension system, it is better to provide a pension system with the return contingent on the amount of the health investment. By doing so, the consumer can raise his utility. However under the presence of the asymmetric information for the health investment, the return is independent of the amount of the health investment at the time when consumers invest their health.

It is well known that this information problem creates the moral hazard problem. Consumers invest their health given the return. However the increase of the health-related investment creates the longevity of their lifetime, that is, their lifetime increases. In this model, the return is determined biologically, depending on the population of the elderly age. Therefore the increase of their lifetime decreases the return. If they consider this demerit of the investment, then the merit to invest their health decreases. In the competitive equilibrium which we discussed in section 5, they invest their health, without considering this demerit. As a result, the level of their investment is excessive, compared with the efficient level. [3] This intuition holds even if there exists money instead of private pension. Therefore the first inefficiency derived here corresponds to the moral hazard problem.

Basic externality effect and Adverse selection

Second, the heterogeneity between communities creates the mobility of money. Therefore the interest rate of money is determined, depending on not only the lifetime in the own community but also that in another community. We can decompose the second inefficiency into two parts, which are (A) the basic externality effect over communities and (B) *the adverse selection* problem by the presence of the different types of consumers in the same economy.[4]

A) The basic externality effect is confirmed by considering the model with two assumptions. The first assumption is that there does not exist the first inefficiency, that is , the consumer consider that the health-related investment affects the interest rate of money in the second period through the longevity of the lifetime. The second one is that two community is symmetric. The second assumption removes the adverse selection problem because all consumers are the same. The first order condition derived under these assumption is in the appendix. We have the following result.

The basic externality effect created by externality to another community through the change on the interest rate by the health-related investment distorts the relative price between health-related investment and consumption. In addition, the basic externality effect creates a distortion only in the economy without the first inefficiency mentioned above.

As mentioned above, the first inefficiency distorts the equation (8), that is, the relative price between health-related investment and consumption. This result means that the basic externality effect also distorts the same relative price as the one distorted by the moral hazard problem. The above result shows that if there exist the first inefficiency, the basic externality effect does not have any distortion. The reason is as follows. Since the community is symmetric, the externality affects only through the marginal effect of the health related investment on the interest rate of money. If there exist the first inefficiency, we do not consider this marginal effect. Therefore the

---

[3] In the economy with one community, Maeda and Akai (1998) proved that the level of health-related investment in a competitive equilibrium is excessive in a competitive equilibrium, compared with the efficient level, without discussing moral hazard problem.

[4] This problem has been pointed out by Rothscild and Stiglits (1976), Wilson (1977), Riley (1979a, 1979b) and Eckstein, Eichenbaum and Peled (1985) in the economy with uncertainty instead of the endogenous longevity.

basic externality effect creates distortion only under the economy without the first inefficiency.

B) Inefficiency created by ***the adverse selection problem*** affects the relative price between the first period's consumption and the second period's consumption. (Compare equation (1) with equation (7).) (The effect of this inefficiency can be confirmed by comparing the simple case where communities are symmetric, with the case developed here. In the simple case, we can derive the equation (27) which is equal to equation (1).)

The introduction of the heterogeneity between communities creates the adverse selection problem. Without loss of generality, community 1 demands the smaller amount of the money than that in community 2, that is, $M^1 < M^2$, which is derived by the difference of the time preference, β. Since the interest rate of money is determined through the demand of money, the increase of money demanded by the agent in community 2 creates negative externality to the agent in community 1. Similarly, the decrease of money demanded by the agent in community 1 affects negative externality to community 2. However the agent in each community does not care this externality effect, which creates ***the adverse selection problem***.

7 The Introduction of Public Pension System

7.1 The introduction of lump sum public pension system

In order to consider the policy scheme to restore the efficient allocation in the competitive equilibrium, we first consider that the government introduces the simple lump-sum pension system into all consumers in both communities. Then the budget constraint in each consumer is written as follows.

$$c_t^i(t) + h^i(t) + l^i(t) + q(t)M^i(t) = w^i - b_t(t)$$

$$c_t^i(t+1) = r(t+1)l^i(t) + q(t+1)M^i(t) + b_t(t+1)$$

where $l^i(t)$ represents private lending and borrowing and $r(t+1)$ represents the interest rate for private lending and borrowing. $b_t(t)$ is the lump sum tax to finance the pension system and $b_t(t+1)$ is the pension payments to the old. Similar to the above section, we only consider the stationary equilibrium such that $b_t(t) = b_0, b_t(t+1) = b_1, c_t^i(t) = c_0^i, c_t^i(t+1) = c_1^i, h^i(t) = h^i, r(t+1) = r$. In this setting, the utility maximization behavior of consumers given the pension system is written as follows;

$$\max u(c_0^i) + \beta^i p(h^i) u(c_1^i)$$

subject to

$$c_0^i + h^i + \frac{c_1^i}{r} = w^i - b_0 + \frac{b_1}{r}.$$

Since the number of young generation is one and the number of old generation is $p(h^i)$, the budget constraint of the government becomes

$$p(h^1)b_1 + p(h^2)b_1 = 2b_0 .$$

Therefore the rate of return of government's pension becomes

$$r^b = \frac{b_1}{b_0} = \frac{2}{p(h^1) + p(h^2)} .$$

Notice that the rate of return of public pension system is not necessarily equal to the rate of return of money. We can characterize a competitive equilibrium (c_0^i, c_1^i, h^i) as follows;

$$\frac{u'(c_0^i)}{u'(c_1^i)} = \beta^i p(h^i) r, \tag{11}$$

$$\frac{u(c_1^i)}{u'(c_0^i)} = \frac{1}{\beta p'(h^i)}, \tag{12}$$

$$c_0^i + h^i + \frac{c_1^i}{r} = w^i + (\frac{r^b}{r} - 1)b_0 . \tag{13}$$

Equations (11) and (12) are the same as the conditions of the competitive equilibrium without any pension system. However equation (13) has changed as long as the interest rate is not equal to the rate of return of the public pension. If the rate of return of pension is larger than the interest rate of money, then the marginal increase of the level of pension increases the lifetime income of consumers. Therefore we have the following proposition.

Proposition 2

Though lump-sum pension system is not enough to achieve the efficient allocation, as long as $r^b > r$ [5], the introduction of this system expands the budget resource of consumers and may increase the utility of agents in both communities and raise efficiency.

Intuition of this proposition is as follows. The lump-sum pension system, financed by the lump-sum tax levied at the same rate in both communities, means that all consumers have to purchase the same amount of the mandatory pension. While the private pension system or money can not force the amount to purchase for consumers, this public pension system can do it. As long as $r^b > r$, the

---

[5] We have

$$\begin{aligned}\frac{1}{r} - \frac{1}{r^b} &= (p(h^i) - p(h^{-i}))\left(\frac{w^1 - c_0^1 - h^1}{(w^1 - c_0^1 - h^1) + (w^2 - c_0^2 - h^2)} - \frac{1}{2}\right) \\ &= (p(h^i) - p(h^{-i}))\left(\frac{M^1}{M^1 + M^2} - \frac{1}{2}\right)\end{aligned} .$$

If the demand for money (saving) is smaller in the community with smaller investment for health, $r^b > r$ holds. In other words, if h^i and M^i is complement, then utility improves. This situation may occur in the plausible case.

public pension system mitigates the adverse selection problem, Therefore the utility increases. In order to confirm the effect of the adverse selection problem, let us consider the symmetric case ($\beta^1 = \beta^2$), where there does not exist the adverse selection problem. Then we have $r^b = r$ and in this setting, the introduction of the lump sum public pension system has no effect, which is the same result as Maeda and Akai (1998) and Samuelson (1958).

7.2 The introduction of local consumption tax

In this subsection, we focus on the consumption tax, instead of the lump sum tax , in order to finance pay-as -you -go pension system. Especially we consider the local consumption tax in the sense that the budget constraint of the pension system is closed in each community. Defining the local consumption tax rate as τ_c^i, the optimization problem is written as follows.

$$\max u(c_0^i) + \beta^i p(h^i) u(c_1^i)$$

subject to

$$(1+\tau_c^i)c_0^i + h^i + \frac{(1+\tau_c^i)c_1^i}{r} = w^i + \frac{b^i}{r}$$

Then the budget constraint of the government becomes

$$\tau_c^i (c_0^i + p(h^i)c_1^i) = p(h^i) b^i$$

Therefore we can characterize an equilibrium as follows;

$$\frac{u'(c_0^i)}{u'(c_1^i)} = \beta^i p(h^i) r, \tag{14}$$

$$\frac{u(c_1^i)}{u'(c_0^i)} = \frac{1}{(1+\tau_c^i)\beta^i p'(h^i)}, \tag{15}$$

$$(1+\tau_c^i)c_0^i + h^i + \frac{(1+\tau_c^i)c_1^i}{r} = w^i + \frac{b^i}{r} \tag{16}$$

$$\tau_c^i (c_0^i + p(h^i)c_1^i) = p(h^i) b^i \tag{17}$$

When the local consumption tax and the pension payment are set such that $\tau_c^i = \frac{1}{1+p'(\bar{h}^i)\bar{c}_1^i} - 1$ and $b^i = \frac{\tau_c^i(\bar{c}_0^i + p'(\bar{h}^i)\bar{c}_1^i)}{p(\bar{h}^i)}$, one of the first order conditions in the efficient allocation (equation (2)) is achieved. However the efficient allocation can not be achieved. Now we have the following proposition.

Proposition 3

The appropriate consumption tax solves the moral hazard problem related to the health-related investment on the interest rate. However, even after the introduction of the consumption tax, the efficient allocation can not be achieved.

Intuition is as follows. In the competitive equilibrium without any tax, there exists the moral hazard problem by externality associated with the health-related investment. When the consumers invest for health, they do not consider the decrease of the interest rate by the increase of the lifetime and the increase of the number of the elderly people. In other words, the relative price of health goods and consumption goods has been distorted by externality. Therefore the introduction of consumption tax removes the distortion associated with the relative price and solves the moral hazard problem.

However there exist two problems in this scheme. One is the rate of the consumption tax. Because of negative externality, the level of investment of consumers is excessive. In order to decrease the investment, the level of the consumption tax which should be introduced becomes negative. Actually the negative consumption tax is difficult to introduce. [The health tax has the similar effect to the local consumption tax. The rate of the health tax becomes positive.] The other is that the consumption tax is not enough to lead the competitive equilibrium to the efficient allocation. There exists the adverse selection problem through the mobility of money over communities based on the heterogeneity of communities. Since the change of the interest rate by the mobility of money distorts the relative price between the first period's consumption and the second period's consumption, the consumption tax equally levied to each period's consumption is not effective. In order to remove inefficiency by the adverse selection problem, we have to introduce another tax, which is the interest tax discussed in the next section.

On the other hand, subtracting b from equations (16) and (17), we have

$$
\begin{aligned}
c_0^i + h^i + \frac{c_1^i}{r} &= w^i + \frac{b^i}{r} - \tau_c^i c_0^i - \frac{\tau_c^i c_1^i}{r} \\
&= w^i + \frac{1}{r}\left(\frac{1}{p(h^i)} - r\right)\tau_c^i c_0^i
\end{aligned}
$$

From the model, we have $\frac{1}{p(h^i)} > r > \frac{1}{p(h^{-i})}$ [6]. Therefore we have the following proposition.

Proposition 4

In the community with smaller investment for health-related goods, its income increases and its utility may increase.

---

[6] We have $rp(h^1) = \frac{p(h^1)(w^1 - c_0^1 - h^1) + p(h^1)(w^2 - c_0^2 - h^2)}{p(h^1)(w^1 - c_0^1 - h^1) + p(h^2)(w^2 - c_0^2 - h^2)}$. Therefore if $p(h^1) < p(h^2)$, then $rp(h^1) < 1$, that is, $\frac{1}{p(h^1)} - r > 0$. Similarly we have if $p(h^1) < p(h^2)$, then $rp(h^2) < 1$, that is, $\frac{1}{p(h^2)} - r < 0$.

The reason why the local consumption tax system affects the lifetime income is related to the adverse selection problem created by the mobility of money. If communities are symmetric (non-adverse selection problem), then we have $r=\frac{1}{p(h)}$, under which the lifetime income becomes equal to that before the introduction of the local consumption tax. This means the consumption tax introduced in order to solve the moral hazard problem has another effect created by the presence of the adverse selection problem.

The reason why the community with smaller investment for health related goods gets a gain is as follows. The local consumption tax is used for the payment to the pension to the elderly people in the second period. Since the elderly people also pay the consumption tax, the consumption tax paid by the elderly people returns them during the same second period, so it does not affect the lifetime income. On the other hand, the consumption tax paid by the young people is transferred to the elderly people. The return from the local consumption tax which the young people pays in the first period , which is defined by $\frac{b^i-\tau_c^i c_1^i}{\tau_c^i c_0^i}$[7], is determined locally while the interest rate of the money is determined in all regions. Therefore the local consumption tax system introduced in the community is not affected by the negative externality created by the money of mobility, that is the adverse selection problem. This community can increase its lifetime income by the introduction of the local consumption tax.

Also the above result suggests that ***as long as the level of the health investment in the community is smaller than that in another community, the marginal increase of the rate of the local consumption tax increase the lifetime income in its community***.

7.3 The introduction of local interest tax

In this subsection, we introduce the local interest tax. Defining the local interest tax rate as τ_r^i, the optimization problem is written as follows.

$$\max u(c_0^i)+\beta^i p(h^i)u(c_1^i)$$

subject to

---

[7] The numerator means the net pension which is equal to the amount of the pension minus the amount of the consumption tax which the elderly people pays in the second period. The denominator means the amount of the consumption tax which the young people pays in the first period. Therefore $\frac{b^i-\tau_c^i c_1^i}{\tau_c^i c_0^i}$ is regarded as the return.

$$c_0^i + h^i + \frac{c_1^i}{r(1-\tau_r^i)} = w^i + \frac{b^i}{r(1-\tau_r^i)}$$

Then the budget constraint of the government becomes

$$r\tau_r^i(w^i - c_0^i - h^i) = b^i$$

Therefore we can characterize an equilibrium as follows;

$$\frac{u'(c_0^i)}{u'(c_1^i)} = \beta^i p(h^i) r(1-\tau_r^i), \tag{18}$$

$$\frac{u(c_1^i)}{u'(c_0^i)} = \frac{1}{\beta^i p'(h^i)} \tag{19}$$

$$c_0^i + h^i + \frac{c_1^i}{r(1-\tau_r^i)} = w^i + \frac{b^i}{r(1-\tau_r^i)} \tag{20}$$

$$r\tau_r^i(w^i - c_0^i - h^i) = b^i \tag{21}$$

When the local health tax is set such that $\tau_r^i = 1 - \frac{1}{p(\bar{h}^i)r}$, one of the first order conditions in the efficient allocation (equation (1)) is achieved.

Contrast to the case of the local consumption tax, the local interest tax can solve the adverse selection problem created by the mobility of money between communities because it adjusts the relative price between two periods. However the moral hazard problem remains.

In addition, subtracting b from equations (20) and (21), we have $c_0^i + h^i + \frac{c_1^i}{r} = w^i$, which is equal to the budget constraint without any tax system.

Proposition 5

The interest tax solves the distortion by free mobility of money between communities. However after the introduction of the interest tax, the efficient allocation can not be achieved.

7.4. The introduction of both consumption tax and interest tax

In this subsection, we finally consider both the local consumption tax and the local interest tax. The optimization problem is written as follows.

$$\max u(c_0^i) + \beta^i p(h^i) u(c_1^i)$$

subject to

$$(1+\tau_c^i)c_0^i + h^i + \frac{(1+\tau_c^i)c_1^i}{r(1-\tau_r)} = w^i + \frac{b^i}{r(1-\tau_r)}$$

Then the budget constraint of the government becomes

$$\tau_c(c_0^i + p(h^i)c_1^i) + r\tau_r(w^i - c_0^i - h^i)p(h^i) = p(h^i)b$$

Therefore we can characterize an equilibrium as follows;

$$\frac{u'(c_0^i)}{u'(c_1^i)} = \mathrm{b}^i p(h^i) r(1-\mathrm{t}^{r}), \tag{22}$$

$$\frac{u(c_1^i)}{u'(c_0^i)} = \frac{1}{(1+\mathrm{t}_c^{\,i})\mathrm{b}^i p'(h^i)}, \tag{23}$$

$$(1+\mathrm{t}_c^{\,i})c_0^i + h^i + \frac{(1+\mathrm{t}_c^{\,i})c_1^i}{r(1-\mathrm{t}_r)} = w^i + \frac{b^i}{r(1-\mathrm{t}_r)} \tag{24}$$

$$\mathrm{t}_c(c_0^i + p(h^i)c_1^i) + r\mathrm{t}_r(w^i - c_0^i - h^i)\,p(h^i) = p(h^i)b \tag{25}$$

When the local consumption tax and the interest tax are set such that $\mathrm{t}_r = 1 - \frac{1}{p'(\bar{h}^i)\bar{r}}$ and $\mathrm{t}_c^{\,i} = \frac{1}{1+p'(\bar{h}^i)\bar{c}_1^i} - 1$, equations (22) and (23) become equal to conditions in the efficient allocation ((1) and (2)). Setting these tax rates, equations (24) and (25) reduces to $c_0^i + h^i + \frac{c_1^i}{r} = w^i$, which is equal to the budget constraint without any tax system. Therefore, by setting the local consumption tax and the interest tax optimally in each community, the efficient allocation is achieved.

Proposition 6

The efficient allocation can be achieved by a pension system financed by local consumption tax and interest tax.

Intuition of this proposition is as follows. In this model, there exist two inefficiencies. First is the moral hazard problem in the sense that consumers does not know that the health-related investment affects the interest rate of money and the pension payment in the second period through the longevity of the lifetime. Second is the adverse selection problem that the heterogeneity between communities affects the interest rate and the demand for money through the mobility of money. The first problem is related to the distorted relative price between consumption and investment. Therefore the consumption tax which affects the relative price is effective to remove this inefficiency. The second problem is related to the relative price between the first period's consumption and the second period's consumption through the interest rate. The interest tax changes its relative price and can solve the second problem.

8. Conclusion

In this paper, with respect to efficiency of the competitive equilibrium when there exist heterogeneous communities, two inefficiencies are derived; first is the moral hazard problem associated with the interest rate of money and second is the adverse selection problem associated with heterogeneity of communities.

In order to improve efficiency, we introduced some public pension systems and examined the effect of each pension system. The main conclusions are as follows. First, we derive the possibility that the lump-sum pension system improves efficiency. Second, the pension system financed by only one type of tax is not enough to achieve the efficient allocation. Finally the introduction of both the local consumption tax and the interest tax can achieve the efficient allocation.

Appendix. Basic externality effect

In order to derive the basic externality effect clearly, we consider the situation where consumers can know the direct reaction to the interest rate of money in the sense that they cannot know the externality effect from/to another community. In addition to it, in order to remove the adverse selection problem created by the heterogeneity, we assume that communities are symmetric. Then we can characterize a competitive equilibrium (c_0, c_1, h) as follows.

$$\frac{u'(c_0)}{u'(c_1)} = \mathrm{b} \tag{26}$$

$$\frac{u(c_1)}{u'(c_0)} = \frac{1 + \frac{p'(h)}{2} c_1}{\mathrm{b} p'(h)} \tag{27}$$

$$c_0 + h + \frac{c_1}{r} = w \tag{28}$$

$$r = \frac{1}{p(h)} \tag{29}$$

As long as a competitive equilibrium exists, the competitive allocation is achieved such that these three equations hold. As shown in equation (26), there does not exist inefficiency created by the adverse selection problem in the symmetric case.

Equation (26) and (28) correspond to equation (3) which is one of the conditions for the efficient allocation. Similarly equation (26) corresponds to equation (1). However equation (27) is not equal to equation (2) because consumers does not consider the externality effect of the health-related investment to another community.

On the other hand, if there exists the moral hazard problem, equation (27) becomes $\frac{u(c_1)}{u'(c_0)} = \frac{1}{\mathrm{b} p'(h)}$ (equation (8)) in spite of the presence of the basic externality effect. Therefore the basic externality effect creates distortion only in the economy with the moral hazard problem. Now, we have the result described in section 6.

References

Davies, J. B. and P. Kuhn, (1992), "Social Security, Longevity, and Moral Hazard," *Journal of Public Economics*, 49, 91-106.

Eckstein, Z., M. Eichenbaum and D. Peled, (1983), "Uncertain lifetimes and the welfare enhancing properties of annuity markets and social security," *Journal of Public Economics*, 26, 303-326.

Maeda, Y. and N. Akai, (1998), "An Optimal Tax Scheme to Finance Social Security in a Model with Endogenous Longevity", *Journal of Economic Research*, 3, 51-67.

Riley, J. G., (1979a), "Informational equilibrium," *Econometrica* 47, 331-359.

Riley, J. G., (1979b), "Noncooperative equilibrium and market signaling," *American Economic Review*, 69, 303-307.

Rothschild, M. and J. Stiglitz, (1976), "Equilibrium in competitive insurance markets: An essay in the economics of incomplete information," *Quarterly Journal of Economics*, 90, 624-649.

Samuelson, P.A., (1958), "An Exact Consumption-Loan Model of Interest with or without the Social Contrivance of Money," *Journal of Political Economy*, 66, 467-482.

Wilson, C., (1977), "A model of insurance markets with incomplete information," *Journal of Economic Theory*, 16, 167-207.