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Central government subsidies to local public goods*

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Abstract. We examine the welfare effects of a central government's subsidy for a local public good in a Nash equilibrium model with two types of public goods. We first show that the welfare effect depends on the substitution and evaluation effects. We also investigate the optimal subsidy rate in a second-best framework and explore how the optimal subsidy scheme depends on the relative evaluation of the two types of public goods.

Key words: Local public good, central public good, subsidy, Lindahl price criterion

JEL Classification: H23, H41, H71

1. Introduction

In a decentralized economy, independent local governments provide local public goods without recognizing possible spillover effects to other regions. This creates efficiency losses due to underprovision of the public good at the Nash solution.[1] Then the central government may consider a subsidy to the local public goods such as matching grants in order to stimulate the production of the local public good. Investigating welfare implication of this subsidy scheme towards efficient public good provision is important.

The standard result for this investigation is given by the Lindahl pricing rule that the sum of matching rates should equal the marginal cost. (For example, Oates

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[1] Gaube (2000) proves that this holds under some plausible conditions.

(1972, Chapter 3 Appendix A)) In an alternative framework in which private charities operate with altruism, Feldstein (1980, 1987), Warr (1982), Roberts (1992), Driessen (1987), Boadway et. al. (1989), Glazer and Konrad (1993), Ihori (1995), Andreoni and Bergstrom (1996) and Kirchsteiger and Puppe (1997) investigate the optimal subsidy level by the central government. Lindahl pricing is shown to be a useful criterion.

However these models have not fully considered the effect on the direct provision of the central government. Actually the central government can directly provide public goods similar to the local public good. Common examples of central government facilitation appear in sectors such as education, health, transportation or public works. It may be natural, due to the budget constraint, that the public good directly provided by the central government decreases when the subsidy scheme increases. If many ministries are responsible for one of the areas of governmental work and the revenue side and the expenditure side are relatively distinct, which is plausible in many countries, the adjustment within one side (revenue side or expenditure side) is executed first. The subsidy scheme, which includes matching grants and the direct provision of public goods, are associated with adjustments in the expenditure side, while the tax scheme is associated with adjustments in the revenue side. It is likely that when matching grants change, public goods would respond first to meet the budget constraint.[2] Therefore, when analyzing the welfare effect of a subsidy, the effect on the central public good should be considered in addition to the effect on tax revenue.

This paper extends this conventional model to a more general one where each local government supplies a local public good with impure spill-over effects to other regions, and the central government directly supplies another type of pure public good in addition to the tax. We adopt a general utility function in which the local public good and the central public good are included separately as imperfect substitutes.

It is shown that the conventional result based on the Lindahl price criterion does not necessarily hold in our generalized framework. In the special case where the central public good is predetermined, the Lindahl price works as a criterion of the optimal subsidizing scheme, although the resulting equilibrium may be inferior due to the sub-optimal provision of the central public good. On the other hand, when the income tax rate is predetermined and the central public good is endogenously determined, the optimal subsidy is not necessarily the same as the Lindahl level. The evaluations of the central public good and the local public good are important to determine the optimal matching grant scheme. Especially it is shown that when the central public good is over-supplied, the local public good should be stimulated by increasing the subsidy rate, more than the level called for by Lindahl pricing.

This paper is organized as follows. Section 2 presents the Nash equilibrium model of local public goods. Section 3 examines the welfare effect of the change in the central government's subsidy to stimulate provision of the local public good.

[2] Another reason for considering the adjustment of public goods first is to take a longer time to change the tax system than public goods. For example, it took more than ten years to introduce the consumption tax and took five years to raise the consumption tax level in Japan.

Section 4 explores the optimal subsidizing scheme of the central government that maximizes social welfare. Section 5 concludes the paper.

2. Model

Consider an economy with n ($\geq$ 2) types of identical regions[3], each of which supplies a local public good that may spill over imperfectly into the other region. In each region, there is a representative agent who cannot move between the regions. The representative agent in each region gets utility from private consumption, c, the amount of the local public good, G, and a centrally provided pure public good, which we call the central public good, hereafter H. Then the utility is represented by the following utility function:

$$U = u(c, G, H), \tag{1}$$

where c, G and H are all assumed to be normal goods. [4] Each local government or the representative individual in each region divides fixed regional income into contributions to the local public good and private consumption. Each local government thus faces the budget constraint

$$c + g = y - T, \tag{2}$$

where g is the quantity of the local public good supplied in each region, y is exogenously given income in each region, and T is the national tax burden. The cost of providing public goods is the same among all regions and is assumed to be unity in terms of consumption goods. Since we assume that the local public good has imperfect spillover effects, G is described as follows;

$$G \equiv g + \lambda(n-1)g, \tag{3}$$

where λ represents the degree of the spillover effect from another region and we assume $(0 \leq \beta \leq 1)$. $\lambda(n-1)g$ becomes the total spillover effects from the other regions.

Next, consider the tax and subsidy policy determined by the central government. The central government controls the national income tax rate, t, and a matching

[3] The general case with heterogeneous regions is examined in Ihori and Akai (1996). Even in this case, similar results are obtained in this general setting as long as the spillover is perfect. As for heterogeneous spillover and heterogeneous productivity, Ihori (1992, 1994) analyzes the effect of the transfer between the heterogeneous agents with the different spillover effects and Ihori (1996) analyzes the effects of the transfer and the changes in the productivity differentials among the heterogeneous agents with productivity differentials. These papers are general in the sense of heterogeneity. However their settings and purposes are completely different because they only consider one type of public goods in the economy and do not incorporate intergovernmental financing.

[4] Even if H is a public input to production in the local sector or the private goods, this general utility function can be justified. First, consider $U = u(c, F(G, H))$, where F is the production function of the good produced by the local sector. Then H is regarded as a pubic input to the production in the local sector. (Glazer and Konrad (1996).) Second, consider $U = u(c, H/n, G)$ where H is a private good. As long as H is allocated equally to each region, H can be regarded as a pure public good with distortion.

grant rate β towards the production of the local public good. $(0 \leq \beta \leq 1)$[5] Then the national tax burden is

$$T = ty - \beta g. \tag{4}$$

Substituting equation (4) into equation (2) gives

$$c + pg = (1-t)y, \tag{5}$$

where p[6] , defined by $1-\beta$, is the effective price of the local public good in each region. The effective price is lowered by the subsidy from the central government. Using equation (3), the above equation can be rewritten as

$$c + pG = (1-t)y + p\lambda(n-1)g. \tag{6}$$

Each local government determines its public good provision and private consumption, treating the effective price of the local public good, p, the income tax rate, t, the spill-over effect from the public good provision by the other regions, $p\lambda(n-1))g$, and the public good provision by the central government, H, as given.[7]

To describe the Nash equilibrium, we define the following expenditure function:[8]

Minimize $E \equiv c + pG$
subject, to $u(c, G, H) \geq U$

The expenditure level of the representative local government is represented as a function of utility, the effective price, and the central public good:

$$E = E(U, p, H).$$

The expenditure function has the following sign and characteristics;

$$E_p = G(U, p, H), G_p < 0,\ c_p > 0,\ c_p = -pG_p{}^9$$

$$E_U = c_U + pG_U > 0,\ G_U > 0,\ c_U > 0,$$
$$G_H \leq 0,\ c_H < 0,\ E_H = c_H + pG_H < 0,$$

[5] $\beta = 1$ indicates the 100 percent subsidy, that is, the local government can supply its public good freely. $\beta = 0$ indicates no subsidy from the central government.

[6] By the definition of p, we get $0 < p \leq 1$. Here $p = 1$ indicates no subsidy from the central government to the local public good.

[7] To get the results clearly, we assume that the non-negativity constraint on providing public goods and consumption goods is not binding for any local government. In other words, we assume interior solutions. Andreoni (1988) and Kirchsteiger and Puppe (1997) investigate the case where at least one agent will not contribute.

[8] As a referee points out, it is easy to understand the intuition of some of results when using the primal approach associated with the standard demand function. This dual approach associated with the expenditure function is useful to analyze the model including externality and is used in many existing papers. (For example, see Ihori (1996).) The welfare analysis can be easily done in the dual approach. In addition, we think that our interpretation of each result by distinguishing two different effects gives clear intuition.

[9] This equation is derived from the property that the compensated demand is homogenous of degree zero in prices.

where $G = G(U, p, H)$ and $c = c(U, p, H)$ represent the compensated demand functions for the local public good and private consumption, respectively.
Using the compensated function and equations (5) and (6), the optimizing behavior of local governments is

$$(1 + \lambda(n-1))c(U, p, H) + pG(U, p, H) = (1-t)(1 + \lambda(n-1))y. \quad (7)$$

The budget constraint of the central government is

$$H = nT.$$

Using equation (2), it can be rewritten as

$$H = ny - nc - ng.$$

Using equation (3) and replacing g with G, we have

$$H = ny - nc(U, p, H) - \frac{n}{1 + \lambda(n-1)} G(U, p, H).$$

Lastly, the basic model reduces to the equations;

$$(1 + \lambda(n-1))c(U, p, H) + pG(U, p, H) = (1-t)(1 + \lambda(n-1))y \quad (8.1)$$

$$H = ny - nc(U, p, H) - \frac{n}{1 + \lambda(n-1))} G(U, p, H). \quad (8.2)$$

Equation (8.1) represents the total budget constraint in overall local governments considering externality effects. Equation (8.2) represents the budget constraint of the central government. If the income tax rate, t, and the subsidy rate, $\beta = 1 - p$, are exogenous, then these two equations determine utility in each region, U and the level of public good provided by the central government, H.

3. Comparative statics

The local public good is likely undersupplied at the Nash solution because each region provides the public good without considering the spillover effect to the other regions. A plausible policy for the central government is to increase the subsidy rate to stimulate the local public good. This section examines the welfare effect of such subsidy policy of the central government.

When we consider the welfare effect in the change of the subsidy rate to the local public good, either the income tax rate or the direct provision of the central public good has to be adjusted to meet the budget constraint of the central government. Namely, when the income tax rate is endogenously adjusted, the provision of the central public good can be regarded as exogenously constant. In such a case, the characteristics of the model are similar to the one without the central public good, that is, $H = 0$, which has been analyzed by Boadway et. al. (1989a). Therefore it is interesting and important to analyze the case where the central public good provision is endogenously adjusted. The endogeneity of the central public good creates new results.

Suppose that any subsidy increase to a local government involves a reduction in the quantity of the central public good. We assume that the national income tax rate is fixed. The welfare effects of an increase in the subsidy rate are derived as follows; (See Appendix A.1 for calculation.)

$$\frac{dU}{d\beta} = -\frac{dU}{dp} = \frac{1}{\Delta^1}\left[n\left(p - \frac{1}{1+\lambda(n-1)}\right)G_p((1+\lambda(n-1))c_H + pG_H)\right.$$
$$\left. +\ (G+\lambda(n-1)c_p)\left(1+nc_H+\frac{n}{1+\lambda(n-1)}G_H\right)\right], \tag{9}$$

where Δ^1, the determinant of the left-hand side of (A-1) in Appendix A.1, has to be positive for the stability condition of Nash equilibrium to be satisfied.[10]

Equation (9) represents the welfare effect when the effective price of the local public good decreases, that is, the matching grant rate increases. This welfare effect contains two terms: first, the substitution effect on the total amount of local public good and second, the evaluation effect, the direct income effect of H from the budget constraint of the central government. (For the sign of each effect, see Table 1.)

Let us explain these two effects intuitively. First, *the substitution effect* means the effect of a decrease in the effective price of the local public good. Since the own substitution effect is always negative, a decrease in p will induce larger g. If $p > \frac{1}{1+\lambda(n-1)}$, the substitution effect increases U. The sign of the substitution effect therefore depends on whether the initial subsidy rate is smaller or larger than the traditional Lindahl level.[11] The intuition is as follows. If the subsidy rate is too small $\left(p > \frac{1}{1+\lambda(n-1)}\right)$, then the substitution effect caused by increasing the subsidy rate has a desirable effect, that is, U increases. In addition, if λ is large or n is large , then we may have $p > \frac{1}{1+\lambda(n-1)}$. If $\lambda = 1$, the critical point becomes 1/n. If $\lambda = 0$, $\frac{1}{1+\lambda(n-1)}$ becomes 1, which means that $p > \frac{1}{1+\lambda(n-1)}$ does not hold and any subsidy reduces welfare.

Second, *the evaluation effect* means the income effect due to a change in the provision of the central public good. Since the functions $c(.)$ and $G(.)$ are compensated demand functions, the absolute value of the derivative of each function with

[10] Though the stability condition does not hold generally, we limit the analysis to the situation where it holds because it is plausible to consider that the equilibrium with the stability exists in the real economy. For example, Δ^1¿0 in the situation where the central public good is over-provided, that is, $nc_H + \frac{n}{1+\lambda(n-1)}G_H > -1$. In addition, even if H is not over-provided, Δ^1¿0 holds in the case with $G_H = 0$ which holds on the following specified utility function, $U = u(c + A(H), G)$.

[11] Due to the optimizing behavior of each region p expresses the direct marginal benefit of G in each region. In the case where the evaluation effect is absent, the sum of direct marginal benefit, $((1+\lambda(n-1))p)$, should be equal to the marginal cost 1, that is, $p = \frac{1}{1+\lambda(n-1)}$, which corresponds to the Lindahl price. This Lindahl price as a criterion of the subsidy scheme has been derived in Oates (1972, chapter 3 Appendix A). However, since Oates (1972) has not considered the change of the central public good, the result obtained in this paper is more general than that by Oates (1972).

Table 1. Main results of comparative statics of change in subsidy on welfare

Case	Substitution effect ΔU	Evaluation effect ΔU
(1) p: decrease (β: increase) H: endogenous t: given	$+*$	$+**$
(2) p: decrease (β: increase) t: endogenous H: given	$+*$	0

Notes

* means that this sign holds if effective price is larger than the Lindahl price, that is,

$$p > \frac{1}{1+\lambda(n-1)}.$$

** means that this sign holds if the substitution effect by a local government is smaller than 1, that is,

$$nc_H + \frac{n}{1+\lambda(n-1)}G_H > -1.$$

respect to the central public good represents evaluations of H in terms of G (and$c$). Put another way, $nc_H + \frac{n}{1+\lambda(n-1)}G_H$ represents compensated changes of private consumption and contributions to local public goods when the central public good marginally increases. In this paper, the marginal cost of the central public good is the same as that of c (and G) and equals one. Therefore, a unit increase of the central public good can save a unit of resource for consumption or contribution to the local public good in the economy as a whole. If $-\left(nc_H + \frac{n}{1+\lambda(n-1)}G_H\right) < 1$, maintaining the utility of a consumer at a constant level means that a one dollar increase in the provision of the central public good can be compensated by reduced spending on the consumption good of less than one dollar, because of diminishing marginal utility from the central public good. In such a case the central public good is regarded as over-provided compared with the local public good. Hence a decrease in H combined with an increase in G increases welfare and regions gain from the evaluation effect.

What is important here is whether the central public good (H) is over-provided or not in the real economy. It seems plausible to consider that the central public good is over supplied. Barro (1990, Chapter 12) suggests that public spending is excessive and hence the substitution effect is smaller than 1 in the real economy.[12] Kormendi (1983) and Aschauer (1985) provide similar empirical evidences. This situation corresponds to $-(nc_H + \frac{n}{1+\lambda(n-1)}G_H) < 1$. Under this realistic situation,

[12] Ihori (1987) and Ihori and Kondo (2000) provide the empirical evidences that the public good is over provided in Japan.

the region could be better off by the additional subsidy policy if the initial subsidy is set by the level of Lindahl price. We have the following result.

Proposition 1 *Suppose the central public good is over-provided and the effective price is set based on Lindahl price. Then, welfare increases by the additional subsidy.*

The intuition of this result is as follows. If $nc + \frac{n}{1+\lambda(n-1)}G > -1$, then H is over-provided compared with the efficient level. An increase of the subsidy rate decreases the central public good H, which raises welfare better from the evaluation effect. Then G should be stimulated more than Lindahl price. Therefore as long as the effective price is initially set based on Lindahl Price, that is, $p = \frac{1}{1+\lambda(n-1)}$ the further increase of the subsidy rate is desirable.[13]

4. Optimal subsidy

This section investigates the subsidy policy that maximizes social welfare. The central government may choose three variables: the subsidy rate, the income tax rate, and the central public good, to maximize social welfare.

We consider the case where the income tax rate is predetermined. This case is also plausible in the actual economy. For example, in the Japanese government, many ministries are responsible for one of the areas of government work; the revenue side and the expenditure side are perfectly distinct. The revenue side, which is associated with tax policy, is planned by Ministry of Finance and the expenditure side is planned by other ministries. It is plausible to assume that expenditure plans are determined by other ministries after the total budget size is given.[14]

4.1. Endogenous supply of the central public good and predetermined income tax rate

Suppose the central government sets the subsidy rate optimally, given the income tax rate. The central public good is supplied to meet the central government budget.

[13] This result is different from the conventional result that Lindahl price is a criterion for welfare improvement. When H is fixed as in Boadway et. al (1989), the evaluation effect is absent. Therefore the welfare effect is directly determined by the substitution effect associated with the effective price.

[14] The following Japanese data supports this separate decision-making. The expenditure share of three main ministries providing public goods (Ministry of Home Affairs, Ministry of Construction and Ministry of Transport) on the total expenditure has not been changed so much for last twenty years. (30.5%(1980), 28.5% (1985), 30.0% (1990), 28.1%(1995), 28.8%(1998)), which is called 'Vertical Administrative' Structure. What Ministry of Finance has done is to adjust the total expenditure size proportionally to meet the total revenue, keeping the expenditure share of other ministries constant. This result suggests that the expenditure details are determined by other ministries separated from Ministry of Finance.

Then from equation (9), the first order condition with respect to p becomes

$$n\left(p-\frac{1}{1+\lambda(n-1)}\right)G_p\left\{(1+\lambda(n-1))c_H+pG_H\right\}$$
$$+(G-\lambda(n-1)pG_p)\left(1+nc_H+\frac{n}{1+\lambda(n-1)}G_H\right)=0. \qquad (10)$$

Defining the compensated price elasticity on G as ε ($\equiv -\frac{pG_p}{G} > 0$), equation (10) can be rewritten as

$$n\left(p-\frac{1}{1+\lambda(n-1)}\right)((1+\lambda(n-1))c_H+pG_H)$$
$$-\left(\frac{1}{\varepsilon}+\lambda(n-1)\right)\left(1+nc_H+\frac{n}{1+\lambda(n-1)}G_H\right)=0, \qquad (11)$$

Noticing that $1+\lambda(n-1))c_H+pG_H<0$ in and $\frac{1}{\varepsilon}+\lambda(n-1)>0$, we have the following result about the optimal matching rates.

Proposition 2 *Suppose the central public good is over-provided,* $(1+nc_H+\frac{n}{1+\lambda(n-1)}G_H \geq 0)$. *Then, the optimal effective price is smaller than the Lindahl price, that is,* $p \leq \frac{1}{1+\lambda(n-1)}$.

The intuition is as follows. When the central public good is regarded as over-provided, substitution of the local public good for the central pubic good should be stimulated by the subsidizing scheme. Therefore an increase in G with a decrease in H is desirable. Since this effect is included as the indirect marginal benefit of the subsidy, the sum of the direct and indirect marginal benefit should be the marginal cost (1). It follows that the direct marginal benefit $((1+\lambda(n-1))p)$ is less than 1 when H is over provided. In other words, the optimal subsidy price is larger than the Lindahl price, that is, $\beta \geq \frac{\lambda(n-1)}{1+\lambda(n-1)}$.[15] Proposition 2 shows that the traditional Lindahl price is not optimal as long as $1+nc_H+\frac{n}{1+\lambda(n-1)}G_H \neq 0$.

From Proposition 2, we have the following policy implication. As Barro (1990) suggests, it is realistic to assume that the central public goods is over-provided. Then the subsidy based on Lindahl pricing is not optimal and the subsidy should be increased.

Example

For further understanding, we consider following two special cases.

$$(a)\ U=u(c,G+A(H))$$

[15] We can derive the optimal effective price from equation (10)', but the general expression is complicated and is not illuminating.

Under this utility function, we get $c_H = 0$. Then, whether $1 + \frac{n}{1+\lambda(n-1)} G_H > 0$ or not determines whether the central public good is over-provided or not. The optimal effective price can be exactly solved as

$$p = \frac{1}{1+\lambda(n-1)} + \frac{1}{n}\left(\frac{1}{G_H} + \frac{n}{1+\lambda(n-1)}\right)\left(\frac{1}{\varepsilon} + \lambda(n-1)\right).$$

We can now consider how the optimal effective price varies with G_H. Namely, the larger is G_H[16], the smaller is the optimal effective price and the larger is the optimal subsidy rate.

The intuition is as follows. An increase in G_H means that the central public good becomes less valuable, compared with the local public good and hence the indirect marginal benefit of an increase in G combined with a decrease in H becomes large. Then it is desirable to stimulate provision by the local government. Therefore the optimal matching rate increases.

$$(b)\ U = u(c + A(H), G)$$

Under this utility function, we get $G_H = 0$. Whether the central public good is over-provided or not corresponds to whether $1 + nc_H > 0$ or not. Now the optimal effective price can be solved as

$$p = \frac{1}{1 + \lambda(n-1) - X}$$

where X is defined as $\frac{(1+nc_H)(\frac{1}{\varepsilon}+\lambda(n-1))}{nc_H}$.

We have the same comparative static results as that in the former special case with respect to in the evaluation of the central public good, c_H. The intuition is similar.

Remark. For comparison with the results obtained in this section, we consider the conventional case where the central government focuses on the subsidy rate, given (or without) the supply level of the central public good. The income tax rate is adjusted to meet the central budget constraint.

In this case, we get the optimality condition associated with the Lindahl price.[17] The reason is as follows. When the central public good is fixed, the evaluation effect is zero. Irrespective of the evaluation of the central public good, the optimal scheme is designed only by considering the spillover effect of the local public good. Therefore the conventional Lindahl criterion holds.

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[16] The increase of G_H means that the compensated value on G of H, that is, the evaluation parameter of the central public good, decreases since $G_H < 0$.

[17] In the case where the central public good is predetermined and the income tax rate is adjusted, the first-order condition of the optimal subsidy rate for utility maximization is

$$\frac{dU}{d\beta} = -\frac{dU}{dp} = \frac{1}{\Delta^2}\left\{(1+\lambda(n-1)y)\left(npG_P - \frac{n}{1+\lambda(n-1)}G_P\right)\right\},$$

which shows that the optimal subsidy rate equals the Lindahl price. (See Appendix A.2 for calculation.)

4.2. Comparison with the first-best case where all policy variables are controlled

It is important to investigate the case where the government can optimally set all policy variables. Suppose now that the central government optimally chooses the rate of income tax as well as the subsidy rate.

If c_H and G_H are elastic and increase dramatically as the income tax rate decreases, there may exist interior solutions such that $\frac{dU}{dt} = 0$. It is then optimal to select the income tax rate such that $nc_H + \frac{n}{1+\lambda(n-1)} G_H = -1$.[18] From Proposition 2, the optimal subsidy rate under this condition becomes $\frac{\lambda(n-1)}{1+\lambda(n-1)}$, which corresponds to the Lindahl price.

If the central government chooses these three variables optimally at the same time, the optimal subsidy is given by the Lindahl price because the government can control H independently of other variables, producing no indirect marginal benefit of G. Then H can be provided efficiently such that $nc_H + \frac{n}{1+\lambda(n-1)} G_H = -1$. [19]

5. Conclusion

It is well known that local public goods that involve spillover benefits to other regions are under-supplied at the Nash solution. This paper examined the welfare effects of a central government's policy instruments when its government provides another public good. We also investigated how the optimal subsidy rates on the local public good are related to the evaluation of private consumption, the local public good, and the central public good.

We showed that the welfare impact depends on the substitution and evaluation effects between the central public good and the local public good. The main results are summarized by Table 1. Suppose the central government subsidizes the regions. Our analysis suggests that such a subsidy policy works well if the central public good has been regarded as over-provided and the initial subsidy rate is smaller than the critical level. In such a case, a decrease in the income tax rate will benefit the regions.

As for the optimal subsidizing scheme, we derived new policy implications. When the central public good is endogenous and the income tax rate is exogenously given, the indirect marginal benefit of an increase in the local public good combines with a decrease in the central public good. Then we have shown that the larger the compensated value of the central public good in terms of the local public good or private consumption, that is, the evaluation of the central public good, the larger is the optimal subsidy rate. Basing the subsidy on the conventional Lindahl pricing

[18] In the case where the subsidy rate is predetermined and the central public good is adjusted, the effect of changes in the income tax rate on utility is

$$\frac{dU}{dt} = \frac{-1}{\Delta^1}(1 + \lambda(n-1)y) \left\{ \left(1 + nc_H + \frac{n}{1+\lambda(n-1)} G_H \right) \right\}.$$

(See Appendix A.2 for calculation.)

[19] If the central government does not have complete information, the Lindahl price cannot be implemented. A complicated tax-subsidy policy would be necessary. See Kirchsteiger and Puppe (1997).

is not necessarily a useful criterion. The evaluation on the central public good is important in efficiently stimulating supply of the local public good.

Appendix

A.1 Endogenous supply of the central public good

Totally differentiating (8.1) and (8.2), we have

$$\begin{bmatrix} (1+\lambda(n-1))c_U + pG_U & (1+\lambda(n-1))c_H + pG_H \\ nc_U + \frac{n}{1+\lambda(n-1)}G_U & 1 + nc_H + \frac{n}{1+\lambda(n-1)}G_H \end{bmatrix} \begin{bmatrix} dU \\ dH \end{bmatrix} = \begin{bmatrix} -\lambda(n-1)c_p - G \\ npG_P - \frac{n}{1+\lambda(n-1)}G_P \end{bmatrix} dp + \begin{bmatrix} -(1+\lambda(n-1))y \\ 0 \end{bmatrix} dt \tag{A.1}$$

where the determinant of $\begin{bmatrix} (1+\lambda(n-1))c_U + pG_U & (1+\lambda(n-1))c_H + pG_H \\ nc_U + \frac{n}{1+\lambda(n-1)}G_U & 1 + nc_H + \frac{n}{1+\lambda(n-1)}G_H \end{bmatrix}$, defined as Δ^1, has to be positive for the Nash equilibrium to be stable.

A.2 Endogenous income tax rate

Totally differentiating (8.1) and (8.2), we have

$$\begin{bmatrix} (1+\lambda(n-1))c_U + pG_U & (1+\lambda(n-1))y \\ nc_U + \frac{n}{1+\lambda(n-1)}G_U & 0 \end{bmatrix} \begin{bmatrix} dU \\ dt \end{bmatrix} = \begin{bmatrix} -\lambda(n-1)c_p - G \\ npG_P - \frac{n}{1+\lambda(n-1)}G_P \end{bmatrix} dp + \begin{bmatrix} -(1+\lambda(n-1))c_H - pG_H \\ -1 - nc_H - \frac{n}{1+\lambda(n-1)}G_H \end{bmatrix} dH \tag{A.2}$$

where the determinant of the matrix in the left hand side, defined as Δ^2, is negative, that is, $\Delta^2 < 0$.

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